

YUKAWA TEXTURES AND HORAVA-WITTEN M-THEORY

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The general structure of the matter Kahler metric in the $\kappa^{2/3}$ expansion of Horava-Witten M-theory with nonstandard embeddings is examined. It is shown that phenomenological models based on this structure can lead to Yukawa and V_{CKM} hierarchies (consistent with all data) without introducing ad hoc small parameters if the 5-branes lie near the distant orbifold plane and the instanton charges of the physical plane vanish. M-theory thus offers an alternate way of describing these hierarchies, different from the conventional models of Yukawa textures.

1 Introduction

Over the past year considerable progress has been made in understanding Horava-Witten heterotic M-theory¹ with “non-standard” embeddings. For a review, see². In this picture, space has an 11 dimensional orbifold structure of the form (to lowest order) $M_4 \times X \times S^1/Z_2$ where M_4 is Minkowski space, X is a 6 dimensional (6D) Calabi-Yau space, and $-\pi\rho \leq x^{11} \leq \pi\rho$. The space thus has two orbifold 10D manifolds $M_4 \times X$ at the Z_2 fixed points at $x^{11} = 0$ and $x^{11} = \pi\rho$ where the first is the visible sector and the second is the hidden sector, each with an a priori E_8 gauge symmetry. In addition there can be a set of 5-branes in the bulk at points $0 < x_n < \pi\rho$, $n = 1 \dots N$ each spanning M_4 (to preserve Lorentz invariance) and wrapped on a holomorphic curve in X (to preserve $N=1$ supersymmetry).

In general, physical matter lives on the $x^{11} = 0$ orbifold plane and only gravity lives in the bulk. The existence of 5-branes allows one to satisfy the cohomological constraints with E_8 on the $x^{11} = 0$ plane breaking to $G \times H$ where G is the structure group of the Calabi-Yau manifold and H is the physical grand unification group. Thus non standard embeddings allow naturally for physically interesting grand unification groups. We consider here the case $G = SU(5)$, and hence $H = SU(5)$.

Phenomenology has played an impor-

tant role in guiding the general structure of Horava-Witten theory. Thus the Calabi-Yau manifold is assumed to have a size $r^{-1} \simeq M_G$ to account for the success of grand unification at $O(10^{16} \text{ GeV})$ and this then requires the orbifold scale to be $(\pi\rho)^{-1} \simeq 10^{15} \text{ GeV}$, to account for the size of the 4D Planck mass. In addition, constructing three generation models has been an important element in string theory from its inception, and recently, three generation models with a Wilson line breaking $SU(5)$ to $SU(3) \times SU(2) \times U(1)$ have been constructed in the M-theory frame work using torus fibered Calabi-Yau manifolds (with two sections)³. Also, the general structure (to the first order) of the Kahler metric of the matter field has been constructed⁴. We examine here this structure and show that it can lead to Yukawa textures with all CKM and quark mass data in agreement with experiment, without any undue fine tuning. Thus M-theory leads to a new way of considering the Yukawa sector of the Standard Model. It is also possible to show that a three generation model with a Wilson line (to break $SU(5)$ to the standard model) and possessing some of the basic properties needed for the phenomenology exists for the Calabi-Yau manifold with del-Pezzo base dP_7 ⁵.

This note is a summary of the above results, and details can be found in⁵

2 Kahler Metric

The bose part of the 11 dimensional gravity multiplet consists of the metric tensor g_{IJ} , the antisymmetric 3-form C_{IJK} and its field strength $G_{IJKL} = 24\partial_{[I}C_{JKL]}$. ($I, J, K, L = 1 \dots 11$). The G_{IJKL} obey field equations $D_I G^{IJKL} = 0$ and Bianchi identities

$$(dG)_{11RSTU} = 4\sqrt{2}\pi\left(\frac{\kappa}{4\pi}\right)^{2/3}[J^0\delta(x^{11}) + J^{N+1}\delta(x^{11} - \pi\rho) + \frac{1}{2}\sum_{n=1}^N J^n(\delta(x^{11} - x_n) + \delta(x^{11} + x_n))]_{RSTU} \quad (1)$$

Here $(\kappa^{2/9})$ is the 11 dimensional Planck scale, and J^n , $n = 0, 1, \dots, N+1$ are sources from orbifold planes and the N 5-branes. These equations can be solved perturbatively in powers of $(\kappa^{2/3})^4$. The effective 4D theory can then be characterized by a Kahler potential $K = Z_{IJ}\bar{C}^I C^J$, Yukawa couplings Y_{IJK} for the matter fields C^I and gauge functions from the physical orbifold plane $x^{11}=0$. To first order, Z_{IJ} takes the form ⁴.

$$Z_{IJ} = e^{-K_T/3} [G_{IJ} - \frac{\epsilon}{2V} \tilde{\Gamma}_{IJ}^i \sum_0^{N+1} (1-z_n)^2 \beta_i^{(n)}] \quad (2)$$

Here $\epsilon = (\kappa/4\pi)^{2/3} 2\pi^2 \rho / V^{2/3}$ is the expansion parameter. V is the Calabi-Yau volume, G_{IJ} , $\tilde{\Gamma}_{IJ}^i$ and Y_{IJK} can be expressed in terms of integrals over the Calabi-Yau manifold ⁴, and K_T is the Kahler potential for the moduli.

3 Phenomenological Yukawa Matrices

If the perturbation analysis is to be a reasonable approximation, the second term of Eq.(2) should be a small correction. A priori one expects G_{IJ} , $\tilde{\Gamma}_{IJ}^i$ and Y_{IJK} to be characteristically of $O(1)$, and the parameter ϵ is not too small. However, the second term will be small if $\beta_i^{(0)}$ were to vanish and if the 5-branes were to be near the distant orbifold plane i.e. $d_n \equiv 1 - z_n$ were small, where $z_n = x_n/\pi\rho$. In the following we will assume

then that

$$\beta_i^{(0)} = 0; \quad d_n = 1 - z_n \cong 0.1 \quad (3)$$

The condition $\beta_i^{(0)} = 0$ is non trivial, but it is possible to show that a three generation model of a torus fibered Calabi-Yau manifold with Wilson like breaking $SU(5)$ to $SU(3) \times SU(2) \times U(1)$ with del -Pezzo base dP_7 has this property⁵.

Eq.(3) then suggests that it is the ϵ term of Eq.(2) that are the third generation contributions to the Kahler metric. A simple phenomenological example for the u and d quark contributions with these properties (and containing the maximum numbers of zeros) is ($f_T \equiv \exp(-K_T/3)$):

$$Z^u = f_T \begin{pmatrix} 1 & 0.345 & 0 \\ 0.345 & 0.132 & 0.639d^2 \\ 0 & 0.639d^2 & 0.333d^2 \end{pmatrix};$$

$$Z^d = f_T \begin{pmatrix} 1 & 0.821 & 0 \\ 0.821 & 0.887 & 0 \\ 0 & 0 & 0.276 \end{pmatrix}. \quad (4)$$

with Yukawa matrices $\text{diag} Y^u = (0.0765, 0.536, 0.585 \exp[\pi i/2])$ and $\text{diag} Y^d = (0.849, 0.11, 1.3)$.

These expressions offer an alternate possibility for generating Yukawa hierarchies. Thus to obtain the physical Yukawa matrices, one must first diagonalize the Kahler metric and then rescale it to unity. Then using the renormalization group equations, one can generate the CKM matrix, and the quark masses. The results are given in the following table:

Quantity	Th. Value	Exp. Value ⁶
$m_t(\text{pole})$	170.5	175 ± 5
$m_c(m_c)$	1.36	1.1-1.4
$m_u(1 \text{ GeV})$	0.0032	0.002-0.008
$m_b(m_b)$	4.13	4.1-4.5
$m_s(1 \text{ GeV})$	0.110	0.093-0.125 ⁷
$m_d(1 \text{ GeV})$	0.0055	0.005-0.015
V_{us}	0.22	0.217-0.224
V_{cb}	0.036	0.0381 ± 0.0021 ⁸
V_{ub}	0.0018	0.0018-0.0045
V_{td}	0.006	0.004-0.013

and $\sin 2\beta=0.31$ and $\sin\gamma=0.97$. The agreement with experiment is quite good. Note also that $m_u/m_d=0.582$ and $m_s/m_d=20.0$ in good agreement with Leutwyler evaluations⁹ 0.553 ± 0.043 and 18.9 ± 0.8 .

While the precise choice of entries in $Z^{u,d}$ and $Y^{u,d}$ are chosen to obtain the above results, as shown in ⁵, the quark mass hierarchies arise naturally from a Kahler metric of the type of Eq. (4). Similarly the smallness of the off diagonal V_{CKM} matrix elements also occur naturally as a consequence of the above model. Thus it is possible for M-theory to generate the Yukawa hierarchies without any undue fine tuning and without introducing ad hoc very small off diagonal entries.

4 Conclusion

Horava-Witten M-theory has now progressed to the point where it offers a fundamental framework for building phenomenological models. Thus it allows for conventional GUT groups (e.g. $SU(5)$, $SO(10)$), accommodates grand unification at $M_G \cong 3 \times 10^{16}$ GeV, and has three generation models where the GUT group breaks to the Standard Model at M_G by a Wilson line.

M theory with non-standard embeddings also offers new possibility of encoding the Yukawa hierarchies in the Kahler metric. This can happen naturally if the 5-branes cluster near the hidden orbifold plane ($d_n \equiv 1 - z_n \simeq 0.1$) and the instanton charges of the physical plane vanish ($\beta_i^{(0)} = 0$). It is possible to construct three generation manifolds possessing these properties⁵. Then if the $\kappa^{2/3}$ term of the Kahler metric is attributed to the third generation of the u-quarks, one can construct models possessing all the experimental hierarchies without any undue fine tuning or ad hoc small parameters. While models of this type are to be viewed as “string inspired” as one can not perform the integrals over the Calabi-Yau manifold, they may give general insights into the nature of the Calabi-

Yau manifold.

5 Acknowledgement

This work was supported in part by NSF grant no. PHY-9722090.

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